

Quantum information transfer in degenerate Raman regime

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Abstract—The interaction of a three level Rydberg atom of Λ -type with a single mode optical field in far off-resonant and at large detuning regimes can be described by an effective degenerate Raman model, where the atomic state can be treated as a two-level system of degenerate states. By means of this approximation, we propose a quantum information transfer of an one-qubit state from a Rydberg atom of Λ -type with degenerate levels to a single-mode field initially in a coherent state.

Index Terms—quantum information, Raman regime.



1 INTRODUCTION

Three-level atoms can appear in different configurations as Λ , Ξ and V , what permits the realization of population inversion with many applications in laser physics [1], [2] and also lasing without inversion. By interacting with an one or two-mode optical field [3], [4], three-level atoms can in some special regimes be effectively described as two-level systems with adiabatic elimination of the highest level in the cases of Λ [5] and Ξ [6] configuration or the adiabatic elimination of the lowest level in the case of V configuration [7].

For a Λ configuration, a special case occurs when the two lower levels are degenerate. In the interaction with a single-mode optical field at far off-resonant and large detuning regimes the atomic states are reduced to a two-level system of degenerate states, characterizing a degenerate Raman regime. This effective interaction leads in fact to an adequate description of the Raman interaction in far off-resonant and large detuning regimes if compared to the full microscopic Hamiltonian of the Raman process [8], in the case short evolving times and de-

scription of physical quantities only involving the square of amplitude probabilities [9].

From quantum optics, the degenerate Raman regime is well known, with important proposes of generation of non-classical states derived from it. For instance, the generation of Fock states [10], [11], superpositions of coherent states [12] and superpositions of phase states [13]. In quantum information, protocols of quantum teleportation were proposed for unknown atomic states [14], unknown entangled coherent states [15] and superpositions of coherent states [16].

In this paper, we propose a quantum information transfer of an one-qubit state from an atom to a single mode field initially in a coherent state. We consider an atom-field interaction in the degenerate Raman regime, such that the lower degenerated states of the atom are involved. Before the interaction, a measurement is realized in one of the atomic degenerate states and a Hadamard gate operation [17] is realized into the single-mode field, realizing finally a quantum information transfer. This type of protocol was developed in other contexts [18], [19], [20], [21], [22], [23], [24], [25] and it was experimentally realized [26], [27]. In our case, the experimental realization can be achieved with three-level Rydberg atoms with principal numbers 49, 50 and 51, adjusting the radiative times, atomic velocities and

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detuning to the single-mode coupling in the situation of degenerate Raman interaction [28], [29], [30], [31], [32], [33]. On the other hand, Hadamard and NOT operations, one-qubit gate operations, can be implemented using linear-optical apparatus [34].

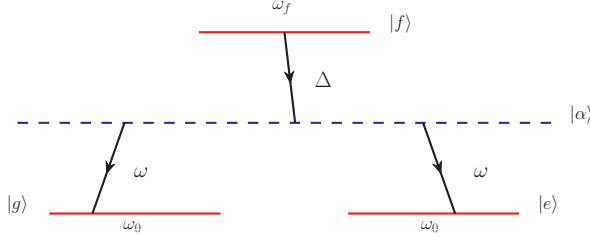


Fig. 1. (Color online) Scheme of degenerate Raman interaction.

We denote the three levels of the three-level Rydberg atom of Λ -type by $|g\rangle$, $|e\rangle$ and $|f\rangle$. Due to the degeneracy, $|g\rangle$ and $|e\rangle$ have same frequency ω_0 . The frequency ω_f is associated to $|f\rangle$. The single-mode field is initially in a coherent state $|\alpha\rangle$ with frequency ω (See figure 1). Under atom-field degenerate Raman coupling the following relation is satisfied

$$\omega_f - \omega_0 = \Delta + \omega, \quad (1)$$

where Δ is the detuning between the atomic transition ($\omega_f - \omega_0$) and the single-mode frequency ω . In the case of large detuning, the upper state $|f\rangle$ can be adiabatically eliminated [4].

In the case of large detuning, short evolving times and physical quantities that only involve the square of the amplitude probabilities, the following effective Hamiltonian can be considered ($\hbar = 1$) [8]

$$\hat{H}_{ef} = \hat{n}\beta(|e\rangle\langle g| + \text{h. c.}), \quad (2)$$

where $\beta = -\lambda^2/\Delta$ is the effective atom-field coupling, λ is the transition coupling from the lower states ($|e\rangle$ and $|g\rangle$) to the upper state ($|f\rangle$), $\hat{n} = \hat{a}^\dagger\hat{a}$ is the number operator, \hat{a} and \hat{a}^\dagger the creation and annihilation operators acting on the single-mode field.

In order to include effects of Stark shifts the term

$$\hat{H}_S = \hat{n}\beta(|g\rangle\langle g| + |e\rangle\langle e|), \quad (3)$$

is added to the effective hamiltonian (2), where we consider, for simplicity, the Stark parameters equal to the effective atom-field coupling β , such that we can write the effective degenerate Raman hamiltonian as [9]

$$\hat{H} = \hat{H}_S + \hat{H}_{ef}. \quad (4)$$

For the single-mode field initially in the coherent state $|\alpha\rangle$, the hamiltonian (4) has validity when the following inequalities [9] are satisfied

$$\Delta^2 \gg 2|2\lambda\alpha|^2, \quad (5)$$

and

$$t \ll \frac{3\Delta^3}{4|\lambda\alpha|^4}. \quad (6)$$

The time evolution during a time t of an initial state of an atom in a superposed state of the form $c_g|g\rangle + c_e|e\rangle$, $|c_g|^2 + |c_e|^2 = 1$, and a field in a coherent state $|\alpha\rangle$ under the interaction (4) is given by

$$\begin{aligned} |\psi(t)\rangle &= (c_+e^{-2i\hat{n}\beta t} - c_-)|g, \alpha\rangle \\ &+ (c_+e^{-2i\hat{n}\beta t} + c_-)|e, \alpha\rangle, \end{aligned} \quad (7)$$

where

$$c_{\pm} = \frac{1}{2}(c_e \pm c_g). \quad (8)$$

We can also write the state (7) as

$$\begin{aligned} |\psi(t)\rangle &= (c_+|e^{-2i\beta t}\alpha\rangle - c_-|\alpha\rangle)|g\rangle \\ &+ (c_+|e^{-2i\beta t}\alpha\rangle + c_-|\alpha\rangle)|e\rangle. \end{aligned} \quad (9)$$

In the case $c_g = 1$, $c_e = 0$, corresponding to evolution of the ground state $|g\rangle$, we have

$$\begin{aligned} |\psi(t)\rangle &= \frac{1}{2}(|e^{-2i\beta t}\alpha\rangle + |\alpha\rangle)|g\rangle \\ &+ \frac{1}{2}(|e^{-2i\beta t}\alpha\rangle - |\alpha\rangle)|e\rangle, \end{aligned} \quad (10)$$

and in the case $c_g = 0$, $c_e = 1$, corresponding to evolution of the excited state $|e\rangle$, we have

$$\begin{aligned} |\psi(t)\rangle &= \frac{1}{2}(|e^{-2i\beta t}\alpha\rangle - |\alpha\rangle)|g\rangle \\ &+ \frac{1}{2}(|e^{-2i\beta t}\alpha\rangle + |\alpha\rangle)|e\rangle. \end{aligned} \quad (11)$$

2 QUANTUM INFORMATION TRANSFER IN DEGENERATE RAMAN INTERACTION REGIME

The single-mode field is initially in a coherent state $|\alpha\rangle$. At degenerate Raman interaction regime, the upper level $|f\rangle$ of the three level Rydberg atom of Λ -type can be neglected, reducing to a two-level system described by the following state

$$|\phi\rangle = c_g|g\rangle + c_e|e\rangle, \quad (12)$$

where

$$|c_g|^2 + |c_e|^2 = 1. \quad (13)$$

In this way, the interaction between the single mode field $|\alpha\rangle$ and the atomic state $|\phi\rangle$ is given by the effective Raman interaction (4).

We want to realize the transfer of the unknown coefficients c_g and c_e from the atom to the single mode field in such a way that in the final step, we will have quantum state transfer from atom to the single mode field. As the atom in the form (12) corresponds to one qubit state, it will correspond to quantum state transfer of one qubit state.

The atom-field interaction occur during a time t , leading the system to achieve the following state

$$\begin{aligned} |\psi_{\alpha\alpha'}\rangle &= (c_+|\alpha'\rangle - c_-|\alpha\rangle)|g\rangle \\ &+ (c_+|\alpha'\rangle + c_-|\alpha\rangle)|e\rangle, \end{aligned} \quad (14)$$

where c_{\pm} is given by (8) and α' is related to α by means of

$$\alpha' = e^{-2i\beta t}\alpha. \quad (15)$$

Measuring the atom in the excited state $|e\rangle$, the field is projected into the state

$$|\phi_+\rangle = c_+|\alpha'\rangle + c_-|\alpha\rangle, \quad (16)$$

and the measurement of the atom in the ground state $|g\rangle$, the field is projected into the state

$$|\phi_-\rangle = c_+|\alpha'\rangle - c_-|\alpha\rangle. \quad (17)$$

We choose the time of interaction $t = \pi/2\beta$ such that from (15) we have

$$|\phi_{\pm}\rangle = c_+|-\alpha\rangle \pm c_-|\alpha\rangle. \quad (18)$$

Now, taking into account the projection relations of coherent states [35]

$$\langle\alpha|\alpha\rangle = 1, \quad (19)$$

$$\langle\alpha|-\alpha\rangle = e^{-2|\alpha|^2}, \quad (20)$$

By choosing $|\alpha|$ sufficiently large, but still satisfying degenerate Raman regime given by the inequalities (6) and (5), we have

$$\langle\alpha|-\alpha\rangle \approx 0. \quad (21)$$

In this case, $|\alpha\rangle$ and $|-\alpha\rangle$ are orthonormal states and the single mode field is described as a two-level system generated by the coherent states $|\alpha\rangle$ and $|-\alpha\rangle$ and capable of storing the qubit state expressed by the atomic state (12).

We note that the coefficients in (18) do not correspond to c_g and c_e , but are related to these by means of relations in (8) or the following

$$c_+ + c_- = c_e, \quad (22)$$

$$c_+ - c_- = c_g. \quad (23)$$

We then realize a Hadamard operation on the field state by means of the action the following operators \hat{A}_α defined by

$$\begin{aligned} \hat{A}_\alpha &= |-\alpha\rangle\langle-\alpha| - |\alpha\rangle\langle\alpha| + |\alpha\rangle\langle-\alpha| \\ &+ |-\alpha\rangle\langle\alpha|, \end{aligned} \quad (24)$$

corresponding to a Hadamard operator [17]

$$\frac{1}{\sqrt{2}}(|0\rangle\langle 0| - |1\rangle\langle 1| + |1\rangle\langle 0| + |0\rangle\langle 1|), \quad (25)$$

where we disconsider the factors of $1/\sqrt{2}$ and make the correspondence $|0\rangle \rightarrow |-\alpha\rangle$ and $|1\rangle \rightarrow |\alpha\rangle$.

The operator \hat{A}_α acts on $|\alpha\rangle$ and $|-\alpha\rangle$ in the following way (see figure 2)

$$\hat{A}_\alpha|\alpha\rangle = -|\alpha\rangle + |-\alpha\rangle, \quad (26)$$

$$\hat{A}_\alpha|-\alpha\rangle = |\alpha\rangle + |-\alpha\rangle, \quad (27)$$

The action of the operator \hat{A}_α on the field $|\phi_+\rangle$ leads single mode field to the following state

$$\hat{A}_\alpha|\phi_+\rangle = c_e|-\alpha\rangle + c_g|\alpha\rangle, \quad (28)$$

corresponding to a quantum information transfer when the atom is detected in the excited

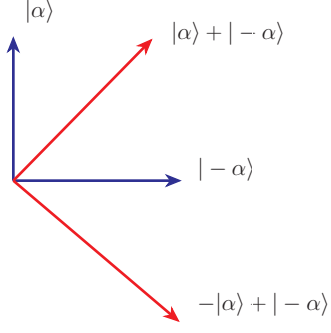


Fig. 2. (Color online) Scheme of $|\alpha\rangle$ and $|- \alpha\rangle$ states (blue arrows) and the transformations under Hadamard operation (red arrows).

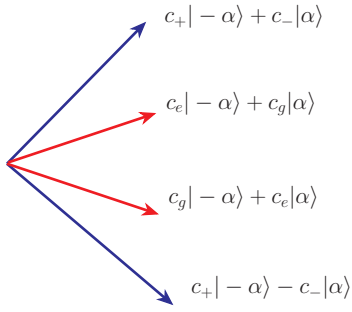


Fig. 3. (Color online) Scheme of $|\phi_+\rangle$ and $|\phi_-\rangle$ states (blue arrows) and the transformations under Hadamard operation (red arrows).

state $|e\rangle$. On the other hand, the action of \hat{A}_α on the field $|\phi_-\rangle$ leads to the state

$$\hat{A}_\alpha|\phi_-\rangle = c_g|-\alpha\rangle + c_e|\alpha\rangle, \quad (29)$$

corresponding to a quantum information transfer when the atom is detected in the ground state $|g\rangle$ (see figure 3).

3 CONCLUSION

In conclusion, we have proposed a protocol of quantum information transfer where the atom-field interacts in degenerate Raman regime, an atomic measurement in any one of the degenerate states is realized and then a Hadamard gate operation is applied into the field, storing in the single-mode field the qubit information present initially in the atom.

Independent of the final state in which the atom is detected (ground $|g\rangle$ or excited state $|e\rangle$), the action of the Hadamard operator \hat{A}_α on

the field after the interaction leads to a quantum information transfer of the coefficients c_g and c_e from the atom to the single-mode field.

The order of the coefficients in (28) and (29) does not matter in our protocol and their interchange could be realized by means of a NOT operation.

Storage of quantum information and its carriage are fundamental problems to the realization of quantum computers, motivating important achievements. In this propose, a simple quantum information transfer was proposed. This situation can be applied in quantum circuits where a qubit comes from an atom qubit state and is stored in a single-mode field qubit state.

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REFERENCES

- [1] V. Vedral, *Modern Foundations of Quantum Optics* (Imperial College Press, London, 2005).
- [2] C. Wei, D. Suter, A. S. M. Windsor, N. B. Manson, *Phys. Rev. A*, **58** (1998) 2310.
- [3] G. R. Agarwal, *Phys. Rev. A*, **1** (1970) 1445.
- [4] C. C. Gerry, J. H. Eberly, *Phys. Rev. A* **42** (1990) 6805.
- [5] Y. Wu, *Phys. Rev. A* **54** (1996) 1586.
- [6] Y. Wu, X. Yang, *Phys. Rev. A* **56** (1997) 2443.
- [7] Z. XinHua, Y. ZhiYong, X. PeiPei, *Sci. China Ser. G Phys Mec. Atron.* **52** (2009) 1034.
- [8] L. Xu, Z-F. Luo, Z-M. Zhang, *J. Phys. B*, **27** (1994) 1649.
- [9] L. Xu, Z-M. Zhang, *Z. Phys. B* **95** (1994) 507.
- [10] A. T. Avelar, T. M. Rocha Filho, L. Losano, B. Baseia, *Phys. Lett. A* **340** (2005) 74.
- [11] L. P. A. Maia, B. Baseia, A. T. Avelar, J. M. C. Malbouisson, *J. Opt. B: Quantum. Semiclass. Opt.* **6** (2004) 351.
- [12] S-B. Zheng, G-C. Guo, *Quantum Semiclass. Opt.* **9** (1997) L45.
- [13] A. T. Avelar, L. A. de Souza, T. M. da Rocha Filho, B. Baseia, *J. Opt. B: Quantum Semiclass. Opt.* **6** (2004) 383.
- [14] S-B. Zheng, G-C. Guo, *Phys. Lett. A*, **232** (1997) 171.
- [15] M. Feng, M. A. S. She, *Comm. Theor. Phys.* **47** (2007) 330.
- [16] S-B. Zheng, *Chin. Phys. Journal*, **42** (2004) 35.
- [17] M. A. Nielsen, I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2000).
- [18] J. I. Cirac, P. Zoller, H. J. Kimble, H. Mabuchi, *Phys. Rev. Lett.*, **78** (1997) 3221.
- [19] Z-B. Feng, Z-L. Cai, C. Zhang, L. Fan, T. Feng, *Optics Communications*, **283** (2010) 3221.
- [20] J. Zhang, K. Peng, S. L. Braunstein, *Phys. Rev. A*, **68** (2003) 013808.
- [21] M. Partenostro, G. M. Palma, M. S. Kim, G. Falci, *Phys. Rev. A*, **71** (2005) 042311.

- [22] C. Di Franco, M. Partenostro, M. S. Kim, Phys. Rev. A, **81** (2010) 022319.
- [23] A.S. Parkins, H. J. Kimble, J. of Opt. B: Quantum Semclass. **1** (1999) 496.
- [24] P-B. Li, Y. Gu, Q-H. Gong, G-C. Guo, Phys. Rev. A, **79** (2009) 042339.
- [25] L. F. Wei, J. R. Johansson, L. X. Cen, S. Ashhab, F. Nori, Phys. Rev. Lett., **100** (2008) 113601.
- [26] D. N. Matsukevich, A. Kuzmich, Science **85** (2004) 306.
- [27] J. F. Sherson, H. Krauter, R. K. Olsson, B. Jusgaard, K. Hammerer, I. Cirac, E. S. Polzik, Nature, **444** (2006) 557.
- [28] S-B.Zheng, G-C. Guo, Phys. Rev. Lett. **85** (2000) 2392.
- [29] X-W. Wang, G-J. Yang, Optics Communications **281** (2008) 5282.
- [30] E. Hagley, et. al., Phys. Rev. Lett., **79** (1997) 1.
- [31] J. M. Raimond, M. Brune, S. Haroche, Rev. Mod. Phys., **73** (2001) 565.
- [32] S. Osnaghi, P. Bertet, A. Auffeves, P. Maioli, M. Brune, J. M. Raimond, S. Haroche, Phys. Rev. Lett. **87** (2001) 037902.
- [33] L. Davidovich, A. Maali, M. Brune, J. M. Raimond, S. Haroche, Phys. Rev. Lett. **71** (1993) 2360.
- [34] K. Koshino, S. Ishizaka, Y. Nakamura, Phys. Rev. A, **82** (2010) 010301.
- [35] D. F. Walls, G. J. Milburn, *Quantum Optics* (Springer-Verlag, Berlin, 2008)